

# Comment on Theory of spin Hall effect

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Chudnovsky[1] recently proposed (arXiv: 0709.0725) a beautifully simple mechanism for the intrinsic spin Hall effect involving an extension of the Drude model. We point out that an equivalent proposal was made by us in an alternative treatment of the problem in Phys.Rev. **B60**, 14787 (1999)[2], reaching identical conclusions. The coincidence of both approaches supports their physical validity. The mechanism applies to the spin Hall effect in all metals and semiconductors and to the anomalous Hall effect in all ferromagnetic metals and semiconductors.

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The theoretical study of spin transport in solids in connection with the spin Hall effect[3, 4] and the anomalous Hall effect of ferromagnetic metals[5] is mired in controversy. Since the original work of Karplus and Luttinger[6] and its refutation by Smit[7], no consensus has been reached on the fundamental question whether or not transverse spin transport can be induced by a perfectly periodic lattice potential in the absence of impurity scattering[8].

Chudnovsky has recently shown[1] that incorporating the spin-orbit interaction in the single particle Hamiltonian and considering the equations of motion within a simple Drude treatment results in a *spin-dependent transverse velocity*

$$\langle \dot{r}_1 \rangle = \frac{\hbar e^2 \tau^2 A}{4m_e^3 c^2} \vec{\sigma} \times \vec{E} \quad (1)$$

where  $\vec{E}$  is the applied longitudinal electric field,  $\tau$  is the relaxation time,  $m_e$  the electron mass, and  $A$  an average of the second derivatives of the crystal potential over the unit cell volume, given in particular for a cubic crystal by

$$A = \langle \frac{\partial^2 \phi}{\partial x^2} \rangle = \frac{4\pi}{3} \frac{Q}{a^3} \quad (2)$$

where  $\phi(x, y, z)$  is the periodic lattice potential resulting from ionic charges  $Q$  on a simple cubic lattice of side length  $a$ .

In Phys.Rev.**B60**, 14787 (1999)[2] we examined the force exerted on a moving spin by the crystal potential by consideration of the equivalent electric dipole[9]

$$\vec{p} = \gamma \frac{\vec{v}}{c} \times \vec{m} \quad (3)$$

where  $\vec{m}$  is the magnetic moment ( $\vec{m} = \mu_B \vec{\sigma}$ ) propagating with drift velocity  $\vec{v}$ , and  $\gamma = (1 - v^2/c^2)^{-1/2} \sim 1$ . This electric dipole interacts electrostatically with the local electric field  $\vec{E}_c$  originating in the crystal potential and experiences a force proportional to the electric field gradient

$$\vec{F}_t = \vec{\nabla}(\vec{p} \cdot \vec{E}_c). \quad (4)$$

The magnitude and direction of this force varies with the position in the unit cell (Fig. 2 of ref.[2]). Upon averaging over particle trajectories[10] we found a net force antiparallel to the electric dipole (i.e. perpendicular to the spin and propagation directions) given by (Eq. 16) of ref.[2])

$$\langle \vec{F}_t \rangle = -\frac{2\pi}{3} \frac{Q}{a^3} \vec{p}. \quad (5)$$

It is easily seen that Chudnovsky's result Eq. (1) is equivalent to Eq. (5), i.e.

$$\frac{m_e \langle \dot{r}_1 \rangle}{\tau} = \langle \vec{F}_t \rangle \quad (6)$$

with  $A$  given by Eq. (2) and the drift velocity  $\vec{v} = (e\tau/m_e)\vec{E}$ .

Eq. (5) can be understood as an 'effective' magnetic field  $B_{eff}$  deflecting the propagating electron just as an applied magnetic field does in the ordinary Hall effect ( $\vec{B}_{eff} = (2\pi/3)(Q/(a^3|e|))\vec{m}$ ). The effect is strong: for example, for a monovalent cubic crystal with  $a = 1\text{\AA}$ , Eq. (5) corresponds to an effective magnetic field  $B_{eff} = 1.94T$ .

In our original proposal of the spin Hall effect[4], we pointed out that it would result from the same mechanism(s) giving rise to the anomalous Hall effect of ferromagnetic metals, and that in particular it would arise from the intrinsic effect discussed here (Ref.3 of ref.[4]). Posteriorly other different mechanisms for intrinsic spin Hall effect were proposed by Murakami et al[11] and by Sinova et al[12] and attracted a lot of attention, applicable to particular lattice structures (e.g. lacking inversion symmetry), band structures and Hamiltonians[8]. In contrast, the mechanism discussed here applies to *all* metals and semiconductors.

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